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# NON-COOPERATIVE TRAFFIC CONTROL IN ATM NETWORKS

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**Summary:** Tranditionally traffic control in communication networks is seeking the optimization of a single network-wide performance objective. Thus, it is assumed that all users and traffic classes in the network have the same objective and cooperate to optimize it. However, in the modern Asynchronous Transfer Mode (ATM) networks, there are multiple users and traffic classes, each with completely different objectives and conflicting quality of service requirements. These different users and traffic classes compete for the limited common resources of the network in order to optimize their own objectives, ignoring the inconvenience that they cause to the others.

We model such traffic control problems in ATM networks with multiple non - cooperative decision-makers on the path flow space. The antagonistic decision-makers act independently but their decisions depend on those of the others since they share the same network resources. We state conditions for existence of the non-cooperative equilibrium and derive the non-cooperative equilibrium conditions. No decision-maker finds beneficial to unilaterally deviate from his strategy, while the others retain theirs. Then, we formulate the non-cooperative equilibrium conditions both as a nonlinear complementarity problem and as a variational inequality problem.

## 1. Introduction

Multimedia applications involving high quality voice, CD sound, video, still images and high speed data traffic are urgently becoming available. Examples of such applications include videoconferencing, transmission of High Definition TeleVision (HDTV), high speed data transmission, remote work on Computer Aided Design (CAD), e.t.c. These multimedia applications require a diversity of multiple types of services with widely varying traffic characteristics and quality-of-service (QOS) requirements. Today the existing multimedia services use separate networks to transmit the voice and the video. A future high-speed network, which is called Broadband - Integrated Services Digital Network (B-ISDN), will have an abundance of bandwidth and will support all these multiple types of services in an integrated way. These different service types will share the same network resources (buffers, switches, transmission lines, e.t.c.) for flexible and efficient resource sharing. This integration of voice, video, data e.t.c. in a common network offers potential cost savings through the sharing of the switching and transmission facilities, flexible internetworking among networks utilizing different transmission media, and enhanced services for multimedia applications. Using a single network to support all these different service types will result in an integrated network planning, management, maintenance, monitoring, security, accounting and billing.

Asynchronous Transfer Mode (ATM) has been accepted as the transport mode for implementing Broadband Integrated Services Digital Networks (B-ISDN). In these networks different traffic classes with widely varying characteristics and conflicting service requirements will be statistically multiplexed and share common switching and transmission resources. It is important to control the traffic and the network resources to provide guaranteed levels of Quality of Service (QOS) for these different traffic classes [1]. In this paper, we control the traffic using admission, routing and load sharing control.

Most previous research assumes a single traffic class and the optimization of a single objective function. However, in an ATM network there are multiple users and traffic classes (each one with possibly different objective function) that compete for the limited common resources of the network. Each decision-maker tries to optimize his own objectives, ignoring the inconvenience that he causes to the others. In this paper, we consider such multiple non-cooperative users and traffic classes with different objectives and formulate the problem both as a Nonlinear Complementarity Problem (NCP) and as a Variational Inequality Problem (VIP). The formulation of the non-cooperative equilibrium conditions characterizing the problem as a NCP or a VIP opens the theory of NCP and VIP for the study and algorithmic solution of it. We also solve specific routing problems with antagonistic decision-makers as Nash games [2,3] and as Stackelberg games [4]. Furthermore, we formulate the resource sharing problem for hierarchical decision-makers (one decision-maker is more powerful than the others) as NCP and VIP [5].

## 2. Queueing Model

In this section, we introduce the analytical model that integrates the admission, routing and load sharing decisions in ATM networks. We formulate the problem on the path flow space, such that the decisions are done at the source nodes. The main purpose of this paper is to introduce the NCP and VIP formulation for non-cooperative traffic control. So, we employ a simple network model rather than a more complete but complex model. The formulation can easily be applied to more complex network models.

Decision-maker $c$ sends traffic to the network for processing and communication. He sends traffic at some source nodes $[s.]$ and requires processing wherever possible in the network. The selection of the destination computer sites, where his traffic is to be processed, constitutes the load sharing decisions. Let that he sends a fraction $\psi^c_{[sd]} \geq 0$ of his traffic to destination $[.d]$. Of cource, the sum of the load sharing fractions from source $[s.]$ to all possible destinations $[.d]$ from this source is equal to one, $\sum_{[.d]} \psi^c_{[sd]} = 1 \quad \forall\ [s.]$.

After deciding which fraction of his traffic will be processed by destination $[.d]$, the decision-maker $c$ must decide which route his traffic will follow to this destination - these are the routing decisions. However, he may decide not to admit some portion of his traffic, for congestion control reasons. So, let that he rejects a fraction $\phi^c_{o[sd]} \geq$

0 and routes a fraction $\phi^c_{\pi[sd]} \geq 0$ of his traffic over a possible path $\pi[sd]$ for the source-destination pair $[sd]$. Of cource, the sum of the rejection fraction and the routing fractions over all possible paths for this source-destination pair is equal to one, $\phi^c_{o[sd]} + \sum\limits_{\pi[sd]} \phi^c_{\pi[sd]} = 1 \quad \forall\ [sd]$.

Let define a single vector for all decision variables for decision-maker $c$, $\mathbf{X}^c = [..., \phi^c_{o[sd]}, ..., \phi^c_{\pi[sd]},\ ..., \psi^c_{[sd]}, ...]^T$, it must hold $\mathbf{X}^c \geq \mathbf{0}$. Define also the vector for all decision-makers: $\mathbf{X} = [\mathbf{X}^1, ..., \mathbf{X}^c,\ ..., \mathbf{X}^C]^T \geq \mathbf{0}$, and the vector for all decision-makers except of decision-maker $c$: $\bar{\mathbf{X}}^c = [\mathbf{X}^1, ..., \mathbf{X}^{c-1},\ \mathbf{X}^{c+1}, ..., \mathbf{X}^C]^T \geq \mathbf{0}$.

In addition, these decision variables must satisfy the conservation equalities: the sum of the admission and routing fractions for all paths $\pi[sd]$ between a given source destination pair $[sd]$ is equal to 1, $\phi^c_{o[sd]} + \sum\limits_{\pi[sd]} \phi^c_{\pi[sd]} = 1 \quad \forall\ [sd]$, and the sum of the load sharing fractions from the source $[s.]$ to every possible destination $[.d]$ from this source is equal to 1, $\sum\limits_{[.d]} \psi^c_{[sd]} = 1 \quad \forall\ [s.]$. Let define the vector $\mathbf{K}^c(\mathbf{X}^c) = [..., \phi^c_{o[sd]} + \sum\limits_{\pi[sd]} \phi^c_{\pi[sd]}, ..., \sum\limits_{[.d]} \psi^c_{[sd]}, ...]^T$, it must hold $\mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$. Define also the vector for all decision-makers: $\mathbf{K}(\mathbf{X}) = [\mathbf{K}^1(\mathbf{X}^1), ..., \mathbf{K}^c(\mathbf{X}^c), ...,\ \mathbf{K}^C(\mathbf{X}^C)]^T = \mathbf{I}$, and for all decision-makers except of decision-maker $c$: $\bar{\mathbf{K}}^c(\bar{\mathbf{X}}^c) = [\mathbf{K}^1(\mathbf{X}^1), ..., \mathbf{K}^{c-1}(\mathbf{X}^{c-1}), \mathbf{K}^{c+1}(\mathbf{X}^{c+1}), ..., \mathbf{K}^C(\mathbf{X}^C)]^T = \mathbf{I}$.

The cost function for decision-maker $c$ over the whole network, $J^c(\mathbf{X})$, is the sum of his cost functions over all network resources. Next, we give some examples of cost functions at a network resource (node, link, computing site, etc.). Throughout this paper, all cost functions are considered to be nonnegative and bounded. Also, the feasibility set is considered to be nonempty. Let the total arrival rate to the network be $\lambda$. Packets from decision-maker $c$ arrive at rate $\lambda^c$ (Poisson) at this resource and require service (general distribution) with mean $1/\mu$ and second moment $\overline{x^2}$.

i) Let model this resource as an $M/G/1$ queue with service rate $C$ and let $\tau$ be a flow independent constant delay (e.g. propagation delay) at this resource. We may take as cost function for decision-maker $c$ at this resource the weighted average packet delay [6]:

$$J^c_{resource} = \frac{\lambda^c}{\lambda} * \left[ \frac{1}{\mu C} + \frac{\sum\limits_k \lambda^k * \overline{x^2} * \mu C}{2 * (\mu C - \sum\limits_k \lambda^k)} + \tau \right]$$

ii) Consider a network resource with $m$ servers, each at service rate $C$. We may take as cost function for decision-maker $c$ at this resource the weighted Erlang's $C$ formula (probability of queueing)

$$J^c_{resource} = \frac{\lambda^c}{\lambda} * P_Q$$

where

$$P_Q = \frac{\dfrac{\left(\dfrac{\sum_k \lambda^k}{\mu C}\right)^m}{m!} * \dfrac{m * \mu C}{m * \mu C - \sum\limits_k \lambda^k}}{\sum\limits_{n=0}^{m-1} \dfrac{\left(\dfrac{\sum_k \lambda^k}{\mu C}\right)^n}{n!} + \dfrac{\left(\dfrac{\sum_k \lambda^k}{\mu C}\right)^m}{m!} * \dfrac{m * \mu C}{m * \mu C - \sum\limits_k \lambda^k}}$$

iii) Consider a network resource with $m$ servers, each at service rate $C$ and no buffers. We may take as cost function for decision-maker $c$ at this resource the weighted Erlang's $B$ formula (or Erlang's loss formula)

$$J^c_{resource} = \frac{\lambda^c}{\lambda} * B(m)$$

where

$$B(m) = \frac{\dfrac{\left(\dfrac{\sum_k \lambda^k}{\mu C}\right)^m}{m!}}{\sum\limits_{n=1}^{m} \dfrac{\left(\dfrac{\sum_k \lambda^k}{\mu C}\right)^n}{n!}}$$

In the optimization problem, we shall use the Lagrangian function, which we define next:

$$\begin{aligned} L^c(\mathbf{X}, \mathbf{Q}^c) \; &= J^c(\mathbf{X}) + \sum_{[sd]} Q^c_{[sd]} * \left[1 - \phi^c_{o[sd]} - \sum_{\pi[sd]} \phi^c_{\pi[sd]}\right] + \\ &+ \sum_{[s.]} Q^c_{[s.]} * \left[1 - \sum_{[.d]} \psi^c_{[sd]}\right] \end{aligned}$$

$$\textit{with } \phi^c_{o[sd]},\ \phi^c_{\pi[sd]},\ \psi^c_{[sd]} \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c.$$

where $\mathbf{Q}^c = [..., Q^c_{[sd]}, ..., Q^c_{[s.]}, ...]^T$ is the vector of decision-maker $c$ multipliers for the constraints of the admission, routing and load sharing fractions. Let also define the vector for all decision-makers: $\mathbf{Q} = [\mathbf{Q}^1, ..., \mathbf{Q}^c, ..., \mathbf{Q}^C]^T$

In the next sections, we consider that the multiple decision-makers compete for the limited network resources and try to use the resources of the network for their own benefit, ignoring the penalty they cause to the other decision-makers. When the decision-makers are in equilibrium, no decision-maker can improve his cost by altering his decision unilaterally. We express the non-cooperative equilibrium conditions as a NCP and a VIP.

## 3. Nonlinear Complementarity Problem Formulation

In this section, we formulate the non-cooperative multi-objective joint problem as a Nonlinear Complementarity Problem (NCP). Let define the gradient vector for the cost function derivatives with respect to the admission, routing and load sharing fractions for all decision-makers:

$$\nabla \mathbf{J}(\mathbf{X}) = \left[ ..., \ \frac{\partial J^c(\mathbf{X})}{\partial \phi^c_{o[sd]}}, \ ..., \ \frac{\partial J^c(\mathbf{X})}{\partial \phi^c_{\pi[sd]}}, \ ..., \ \frac{\partial J^c(\mathbf{X})}{\partial \psi^c_{[sd]}}, \ ... \right]^T$$

**Theorem 1:**

*Consider the multi-objective joint admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers. If for each decision-maker $c$, $J^c$ is differentiable and convex in $\mathbf{X}^c \geq \mathbf{0}$, $\mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$ for each fixed value of $\bar{\mathbf{X}}^c \geq \mathbf{0}, \bar{\mathbf{K}}^c(\bar{\mathbf{X}}^c) = \mathbf{I}$, then $\mathbf{X}^* \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^*) = \mathbf{I}$ is a non-cooperative equilibrium if and only if it solves the following Nonlinear Complementarity Problem:*

$$\begin{gathered}
[\nabla \mathbf{J}(\mathbf{X}^*) - \mathbf{Q} \quad \mathbf{I} - \mathbf{K}(\mathbf{X}^*)]^T * [\mathbf{X}^* \quad \mathbf{Q}] = \mathbf{0} \\
[\nabla \mathbf{J}(\mathbf{X}^*) - \mathbf{Q} \quad \mathbf{I} - \mathbf{K}(\mathbf{X}^*)] \geq \mathbf{0} \\
[\mathbf{X}^* \quad \mathbf{Q}] \geq \mathbf{0}
\end{gathered}$$

Proof: For each decision-maker $c$, we have the following problem

$$\begin{array}{ll}
\textit{minimize} & J^c(\mathbf{X}^{1*}, ..., \mathbf{X}^c, ..., \mathbf{X}^{C*}) \\
\textit{with respect to} & \mathbf{X}^c \\
\textit{such that} & \mathbf{X}^c \geq \mathbf{0}, \ \bar{\mathbf{X}}^{c*} \geq \mathbf{0}, \ \ \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}, \ \bar{\mathbf{K}}^c(\bar{\mathbf{X}}^{c*}) = \mathbf{I}
\end{array}$$

The Lagrangian for each decision-maker $c$ is

$$L^c = J^c + \sum_{[sd]} Q^c_{[sd]} * \left[ 1 - \phi^c_{o[sd]} - \sum_{\pi[sd]} \phi^c_{\pi[sd]} \right] + \sum_{[s.]} Q^c_{[s.]} * \left[ 1 - \sum_{[.d]} \psi^c_{[sd]} \right]$$

with $\phi^c_{o[sd]}, \ \phi^c_{\pi[sd]}, \ \psi^c_{[sd]} \geq 0 \quad \forall \ \pi[sd], \ [sd], \ c$

The cost function for each decision-maker $c$, $J^c$, is convex in $\mathbf{X}^c \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$, so the Karush-Kuhn-Tucker necessary conditions are also sufficient:

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \phi^c_{o[sd]}} * \phi^{c*}_{o[sd]} = 0 \ \Rightarrow \ \left[\frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}\right] * \phi^{c*}_{o[sd]} = 0 \quad \forall \ [sd], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \phi^c_{\pi[sd]}} * \phi^{c*}_{\pi[sd]} = 0 \ \Rightarrow \ \left[\frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}\right] * \phi^{c*}_{\pi[sd]} = 0 \quad \forall \ \pi[sd], \ [sd], c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \psi^c_{[sd]}} * \psi^{c*}_{[sd]} = 0 \ \Rightarrow \ \left[\frac{\partial J^c(\mathbf{X}^*)}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}\right] * \psi^{c*}_{[sd]} = 0 \quad \forall \ [.d], \ [s.], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \phi^c_{o[sd]}} \geq 0 \ \Rightarrow \ \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]} \geq 0 \qquad \forall \ [sd], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \phi^c_{\pi[sd]}} \geq 0 \ \Rightarrow \ \frac{\partial J^c(\mathbf{X}^*, \mathbf{\Psi}^*)}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]} \geq 0 \ \ \forall \ \pi[sd], \ [sd], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial \psi^c_{[sd]}} \geq 0 \ \Rightarrow \ \frac{\partial J^c(\mathbf{X}^*)}{\partial \psi^c_{[sd]}} - Q^c_{[s.]} \geq 0 \qquad \forall \ [.d], \ [s.], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q}^c)}{\partial Q^c_{[sd]}} = 0 \ \Rightarrow \ \phi^{c*}_{o[sd]} + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]} = 1 \ \ \forall \ [sd], \ c$$

$$\frac{\partial L^c(\mathbf{X}^*, \mathbf{Q})}{\partial Q^c_{[s.]}} = 0 \ \Rightarrow \ \sum_{[.d]} \psi^{c*}_{[sd]} = 1 \qquad \forall \ [s.], \ c$$

$$\phi^{c*}_{o[sd]}, \ \phi^{c*}_{\pi[sd]} \geq 0 \qquad \forall \ \pi[sd], \ [sd], \ c$$

$$\psi^{c*}_{[sd]} \geq 0 \qquad\qquad \forall \ [.d], \ [s.], \ c$$

After some algebraic manipulations, we find that the NCP as defined in the Theorem is equivalent to the Karush-Kuhn-Tucker necessary and sufficient conditions. □

**Theorem 2:** <u>*existence*</u>
*Consider the multi-objective joint admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers.*
*If* $[\nabla \mathbf{J}(\mathbf{X}) - \mathbf{Q} \quad \mathbf{I} - \mathbf{K}(\mathbf{X})]$ *is differentiable in* $[\mathbf{X} \quad \mathbf{Q}] \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I}$
*and its Jacobian matrix is strongly copositive in* $[\mathbf{X} \quad \mathbf{Q}] \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I}$
*then there exists a non-cooperative equilibrium.*

Proof: It is known [7], that the NCP: $x * f(x) = 0; \ \ f(x) \geq 0; \ \ x \geq 0$ has a solution if $f$ is differentiable in $E^n_+$ and its Jacobian matrix is strongly copositive in $E^n_+$. □

## 4. Variational Inequality Problem Formulation

In this section, we formulate the non-cooperative multi-objective problem as a Variational Inequality Problem (VIP).

**Theorem 3:**
*Consider the multi-objective join admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers.*

*If for each decision-maker c, $J^c$ is continuously differentiable and convex in $\mathbf{X}^c \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$ for each fixed value of $\bar{\mathbf{X}}^c \geq \mathbf{0}, \bar{\mathbf{K}}^c(\bar{\mathbf{X}}^c) = \mathbf{I}$,*

*then $\mathbf{X}^* \geq \mathbf{0}, \mathbf{K}(\mathbf{X}^*) = \mathbf{I}$, is a non-cooperative equilibrium if and only if it solves the following Variational Inequality Problem:*

$$\nabla \mathbf{J}(\mathbf{X}^*) * (\mathbf{X} - \mathbf{X}^*) \geq \mathbf{0} \qquad \forall\ \mathbf{X} \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I},$$

Proof: If $\mathbf{X}^{c*}$ is a local minimum for the following minimization problem

$$\begin{array}{ll} \textit{minimize} & J^c(\mathbf{X}^{1*}, ..., \mathbf{X}^c, ..., \mathbf{X}^{C*}) \\ \textit{with respect to} & \mathbf{X}^c \\ \textit{such that} & \mathbf{X}^c \geq \mathbf{0},\ \bar{\mathbf{X}}^{c*} \geq \mathbf{0},\ \ \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I},\ \bar{\mathbf{K}}^c(\bar{\mathbf{X}}^{c*}) = \mathbf{I} \end{array}$$

and $J^c$ is a continuously differentiable convex function over the nonempty convex, closed and bounded set $\mathbf{X} \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I}$, then

$$\sum_{[sd]} \left\{ \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}} * (\phi^c_{o[sd]} - \phi^{c*}_{o[sd]}) \ + \sum_{\pi[sd]} \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{\pi[sd]}} * (\phi^c_{\pi[sd]} - \phi^{c*}_{\pi[sd]}) + \right.$$

$$\left. + \ \frac{\partial J^c(\mathbf{X}^*)}{\partial \psi^c_{[sd]}} * (\psi^c_{[sd]} - \psi^{c*}_{[sd]}) \right\} \ \geq 0 \quad \forall\ \mathbf{X}^c \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I},\ c$$

Summing over all decision-makers

$$\sum_c \sum_{[sd]} \left\{ \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}} * (\phi^c_{o[sd]} - \phi^{c*}_{o[sd]}) \ + \sum_{\pi[sd]} \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{\pi[sd]}} * (\phi^c_{\pi[sd]} - \phi^{c*}_{\pi[sd]}) + \right.$$

$$\left. + \frac{\partial J^c(\mathbf{X}^*)}{\partial \psi^c_{[sd]}} * (\psi^c_{[sd]} - \psi^{c*}_{[sd]}) \right\} \geq 0 \qquad \forall\ \mathbf{X} \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I}$$

□

Another equivalent VIP formulation is given in the following Theorem:

**Theorem 4:**

*Consider the multi-objective joint admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers. If for each decision-maker c, $J^c$ is continuously differentiable and convex in $\mathbf{X}^c \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$, for each fixed value of $\bar{\mathbf{X}}^c \geq \mathbf{0}$, $\bar{\mathbf{K}}^c(\bar{\mathbf{X}}^c) = \mathbf{I}$, then $\mathbf{X}^* \geq \mathbf{0}, \mathbf{K}(\mathbf{X}^*) = \mathbf{I}$, is a non-cooperative equilibrium if and only if it solves the following Variational Inequality Problem:*

$$[\nabla \mathbf{J}(\mathbf{X}^*) - \mathbf{Q} \quad \mathbf{I} - \mathbf{K}(\mathbf{X}^*)] * [\hat{\mathbf{X}} - \mathbf{X}^* \quad \hat{\mathbf{Q}} - \mathbf{Q}] \geq 0 \qquad \forall\ [\hat{\mathbf{X}} \ \ \hat{\mathbf{Q}}] > \mathbf{0}$$

Proof:

Karamardian [8] shows that the NCP: $f(x^*) * x^* = 0; \quad f(x^*) \geq 0; \quad x^* > 0$ and the VIP: find $x^*$ such that $f(x^*) * (x - x^*) \geq 0 \ \ \forall x > 0$ are equivalent.□

**Theorem 5:** *existence*

*Consider the multi-objective joint admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers.*
*If $\nabla \mathbf{J}(\mathbf{X})$ is continuous on $\mathbf{X} \geq \mathbf{0}, \mathbf{K}(\mathbf{X}) = \mathbf{I}$ and bounded,*
*then there exists a non-cooperative equilibrium.*

Proof: For a VIP: find $x^*$ such that $f(x^*) * (x - x^*) \geq 0 \ \ \forall x \in K$, if $f$ is continuous on $K$, then there exists a $x^*$ that solves VIP [9]. □

The following Theorem follows from either the NCP or the VIP formulation.

**Theorem 6:**

*Consider the multi-objective joint admission, routing and load sharing problem in networks with multiple non-cooperative decision-makers. If for each decision-maker $c$, $J^c$ is differentiable and convex in $\mathbf{X}^c \geq \mathbf{0}, \mathbf{K}^c(\mathbf{X}^c) = \mathbf{I}$, for each fixed value of $\bar{\mathbf{X}}^c \geq \mathbf{0}$, $\bar{\mathbf{K}}^c(\bar{\mathbf{X}}^c) = \mathbf{I}$, then $\mathbf{X}^* \geq \mathbf{0}, \mathbf{K}(\mathbf{X}^*) = \mathbf{I}$, is a non-cooperative equilibrium solution if and only if*

*admission control* $\forall [sd], \ c$

$$\phi^{c*}_{o[sd]} > 0 \ only \ if \qquad \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}} \leq \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{\pi[sd]}} \ \ \forall \ \pi[sd]$$
$$\phi^{c*}_{o[sd]} = 0 \qquad o.w.$$

*routing* $\forall \ \pi[sd], \ [s.], \ c$

$$\phi^{c*}_{\pi[sd]} > 0 \ only \ if \qquad \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{\pi[sd]}} = \min \left\{ \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{o[sd]}}, \min_{p[sd]} \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{p[sd]}} \right\}$$
$$\phi^{c*}_{\pi[sd]} = 0 \qquad o.w.$$

*load sharing* $\forall [sd], \ c$

$$\psi^{c*}_{[sd]} > 0 \ only \ if \qquad \frac{\partial J^c(\mathbf{X}^*)}{\partial \psi^c_{[sd]}} \leq \frac{\partial J^c(\mathbf{X}^*)}{\partial \phi^c_{[sd']}} \ \ \forall \ [sd']$$
$$\psi^{c*}_{[sd]} = 0 \qquad o.w.$$

$$\phi^{c*}_{o[sd]} + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]} = 1 \quad \forall\ [sd],\ c$$

$$\sum_{[.d]} \psi^{c*}_{[sd]} = 1 \qquad\qquad \forall\ [s.],\ c$$

## 4. Conclusions

The usual approach to traffic control problems in networks is the optimization of a single objective function as seen by the network administrator. However, in an ATM network there are multiple users and traffic classes each one with possibly different objective function. We consider such multiple decision-makers that compete for the limited common resources of the network, in order to optimize their own objectives. Each decision-maker tries to optimize his own performance objective controlling his admission, routing and load sharing decision variables.

We state conditions for existence of the non-cooperative equilibrium and derive the non-cooperative equilibrium conditions. When the decision-makers are in equilibrium, no one has a rational motive to unilaterally deviate from his equilibrium strategy. Then we express these conditions as a nonlinear complementarity problem and as variational inequality problem.

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